root of the energy-flux is proportional to the *amplitude of the electric vector* in the light; and this leads to the view that the rate of ionization is in proportion to the maximum value of the electric force operating on the atoms of selenium. This seems probable enough. Too much stress should not, perhaps, be laid at present on the quantitative aspect of this result. The law $C-C_0 \propto I^{\frac{1}{4}}$ or $(C-C_0)^2 \propto I^{\frac{1}{2}}$ has been found to hold for white light, in which, of course, the amplitude of the electric vector has, at any moment, a variety of values corresponding to the various frequencies; and there is no single value which can be deduced from the total intensity simply by taking the square root. Moreover, it is not just one wave-length which affects selenium; the effect is distributed more or less throughout the spectrum. Further investigation is required in this connexion; and it has already been found that the same law—viz., $C-C_0 \propto I^{\frac{1}{4}}$,—holds when the light red has been rendered more nearly monochromatic by using a filter permitting the passage of 6300 to 7000 A.U. only. Apart from the exact quantitative relation, however, the fact remains that the rate of production of ions is not proportional to the incident flux of energy. And this at least suggests that the effect of light on selenium is in the nature of a "trigger" action rather than the mere transformation of one kind of energy into another.

Imperial College.
Jan. 30, 1920.

XLVIII. *On Gravitation. Theoretical and Experimental Researches.* By Q. MAJORANA, *Professor of Physics in the Polytechnic School of Turin (Italy)**.

ORIGIN of the Researches.—In a preceding paper † on the theory of relativity and on the influence of the movement of the source or of a mirror on the propagation of light, I expressed the doubt whether among other unknown causes that might have some influence on the phenomenon, there might be the gravitational field of the earth. Without pretending to connect now two orders of phenomena so different, I shall give an account in this paper of some recent experiments of mine on gravitation that have their origin in the ones formerly described.

* Communicated by the Author. Translated from the Italian by A. Lion.

† Phil. Mag. xxxv. p. 163 (1918) and xxxvii. p. 145 (1919).

Characters of Newton's Law.—In its simplicity this law seems the most perfect of the physical laws. Up to now no influence of the nature of the medium has been detected in the propagation of the attractive force between two material masses. The researches of Austin and Thwing *, Kleiner †, Laager ‡, Cremieu §, Erisman ‖, and others having in view the discovery of an action of that kind, failed utterly. In consequence of Laager's experiment, in which he studied the weight of a silver ball, covered with a thick coat of lead, it may be thought that the lack of effect has been until now verified up to the approximation of about 5.10^{-5}. The result was obviously a confirmation of the exactitude of Newton's Law.

Doubts on the exactitude of Newton's Law.—It does not seem right to me to deduce from an experiment, like Laager's for instance, that what has been verified in the Laboratory, might repeat itself with the same appearance, even in astronomical cases. Hence it would not be right to deduce therefrom that the mass of the silver ball would appear the same if in the centre of the earth, or in the centre of the sun (333,000 times the mass of the earth).

Let us admit as an hypothesis, that the mass may appear smaller if surrounded by other masses, that is, that there is a diminution of the force of gravitation on account of its propagation across a material medium. This diminution might be owing to a property of the material medium, to be compared with electric or magnetic permeability, or to the progressive absorption of the force. In the first case, if the analogy with the electrical and magnetical phenomena could be proved, small thicknesses of the medium would be sufficient to allow of the verification of the hypothetical permeability of gravitation; this has not been done in the experiments known up to now. In the second case the absorption would occur for very great thicknesses of medium, and therefore escape the researches of the laboratory, and yet manifest itself in the celestial bodies. In consequence, this second hypothesis of absorption seems more probable, and its conception would be more easy, if the force of gravitation could be explained by a kind of energical flux, continually emanating from ponderable matter. This flux

* Phys. Rev. v. (1897).

† *Arch. sc. phys. et nat.* xx. p. 420 (1905).

‡ *Dissert. Zürich* (1904).

§ *C. R.* cxl. p. 80 (1905); cxli. pp. 653, 713 (1905); cxliii. p. 887 (1906).

‖ *Vierteljahrschr.* liii. p. 157 (1908).

would be gradually absorbed, as in the case of light in its propagation through a not perfectly transparent medium: Newton's Law would only be exact in the first approximation.

Consequences of the Hypothesis of Absorption.—The first consequence would be the knowledge of the *true mass*, and of the *apparent mass.* The true mass would be that property of the mass to which the attractive force is proportional, if it is reduced in very small particles. The apparent mass is, on the other hand, the apparent value that the true mass assumes in consequence of the gradual absorption.

In order to respect the principle of the conservation of energy, it would be required, furthermore, to admit that any kind of matter is gradually transforming itself. This would be analogous, so to speak, to what happens with radium, with the difference that for this substance the transformation lasts some thousands of years, whilst for every one of the other known substances, the transformation would last enormously longer.

Another consequence might be deduced: since the force of gravitation is produced by an energical flux that has been absorbed, and as the energy cannot be destroyed, it ought to transform itself, into heat, for instance. Therefore matter subjected to gravitation is heated; this fact would give a new explanation of all the solar heat or of a part of it at least. Furthermore, the hypothesis might be kept in mind when considering the fact, which is nearly certain, that there are no *non-luminous* stars of great dimensions in the sky; ponderable matter ought to heat itself if condensed into little space.

It is, however, with the utmost reserve that I state the hypothesis of the energetic character of the force of gravitation; and if considerations that I do not detect at present should arise in the future and make me judge it erroneous, I should withdraw it.

I rather consider as probable, and in fact, after some experiments that I shall describe as certain, the hypothesis of gravitational absorption.

Analytical Researches.—In order to be able to establish a plan of experiments with the purpose of verifying the hypothesis of absorption, it is necessary to consider this hypothesis analytically. A physical quantity, *sui generis*, can now be defined and called *flux of gravitational action*; it is not yet necessary to connect the notion of this quantity with the notion of energy. Let us suppose a material particle dm so small that its inner gravitational absorption might be valued as null. We can suppose, in consequence of the above stated hypothesis, that it would continually put forth a certain flux, proportional to dm, that is to say, $k\,dm$, uniformly irradiated in all directions. Let us suppose the particle to be in a vacuum; across a solid angle subtending the surface $d\omega$ at the distance of 1, the flux would only be

$$\phi = k\frac{dm\,d\omega}{4\pi}.$$

If the particle instead of being in a vacuum is in a medium of true density δ_v, the flux that will have arrived at the distance x from the particle, will be expressed by

$$\phi = k\frac{dm\,d\omega}{4\pi}e^{-Hx}. \quad \ldots \quad (1)$$

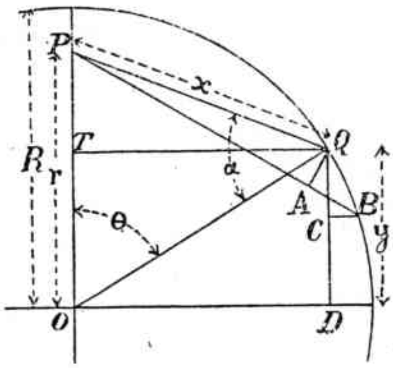
This is equivalent to admitting a gradual absorption of the flux, proportional to its value at every point, to the thickness of the medium that has been crossed, and to the medium's density. It is supposed, in fact, that

$$H = h\delta_v, \quad \ldots \quad (2)$$

H being the *quenching factor* for the density δ_v, and h the quenching factor for density 1.

We will now consider a massive sphere, with the uniform density δ_v; and determine the flux that emerges from it. Let us call R its radius, O its centre (fig. 1). I consider an

Fig. 1.



inner point P of it, in which the mass dm would be concentrated. I trace the PO radius of the sphere passing through P; and I trace QPB, an infinitesimal angle, with the vertex at P; I trace QA perpendicular to PQ. I say: OP$=r$; PQ$=x$; QD$=y$. I make the triangle QPA turn around the PO axis; the AQ segment will trace the $2\pi . TQ . QA$ surface. It is possible in equation (1) to substitute for $d\omega$ this surface, reduced to unit distance from P, that is to say, dividing it by x^2. We have

$$\phi = k \frac{dm \, TQ . QA}{2x^2} e^{-Hx}.$$

QD is to be traced parallel to OP; B projected normally to QD on c; we have $Qc = dy$. I call $P\hat{Q}O = \alpha$; $P\hat{O}Q = \theta$; it appears from the figure that

$$dy = QB \sin\theta \; ; \; QA = QB \cos\alpha \; ;$$

therefore $$QA = \frac{dy}{\sin\theta} \cos\alpha.$$

Moreover,

$$TQ = R \sin\theta \; ; \; x = \sqrt{R^2 + r^2 - 2ry}.$$

Differentiating

$$dy = -\frac{x\,dx}{r}.$$

From the triangle OPQ we have

$$r^2 = x^2 + R^2 - 2xR \cos\alpha,$$

therefore $$\cos\alpha = \frac{x^2 + R^2 - r^2}{2xR}.$$

Hence we have

$$\phi = -k \frac{dm}{4r} \left(1 + \frac{R^2 - r^2}{x^2}\right) e^{-Hx} dx.$$

Let us call dF the total flux of action that is put forth by the dm particle, and that succeeds in emanating itself from the sphere; this flux will be given by the integral of ϕ, taken between $R+r$ and $R-r$ limits,

$$dF = -k \frac{dm}{4r} \int_{R+r}^{R-r} \left(1 + \frac{R^2 - r^2}{x^2}\right) e^{-Hx} dx \; ;$$

and carrying out the integration

$$dF = k \frac{dm}{4r} \left[\frac{e^{-Hx}}{H} + \frac{(R^2 - r^2)}{x} e^{-Hx} + H(R^2 - r^2) \int \frac{e^{-Hx}}{x} dx \right]_{R+r}^{R-r}$$

Extending up to the limits, where it is possible,

$$dF = k\frac{dm}{4r}\left[e^{-H(R-r)}\left(\frac{1}{H}+R+r\right) - e^{-H(R+r)}\left(\frac{1}{H}+R-r\right) - H(R^2-r^2)\int_{R-r}^{R+r}\frac{e^{-Hx}}{x}dx\right].$$

The integral remaining in this expression is transcendent, and one can only obtain its value by developing this expression in series; but this can be avoided by an opportune artifice. Let us call dm not only the mass that is contained in the point P, but all the mass of a spherical layer whose radius is r and thickness dr,

$$dm = 4\pi r^2 \delta_v dr,$$

therefore

$$dF = k\pi\delta_v r\,dr\left[e^{-H(R-r)}\left(\frac{1}{H}+R+r\right) - e^{-H(R+r)}\left(\frac{1}{H}+R-r\right) - H(R^2-r^2)\int_{R-r}^{R+r}\frac{e^{-Hx}}{x}dx\right].$$

To obtain the value of the total flux that emerges from all the points of the sphere, it is necessary to integrate this expression from O to R, and we have

$$F = k\pi\delta_v\int_0^R r\,dr\left[e^{-H(R-r)}\left(\frac{1}{H}+R+r\right) - e^{-H(R+r)}\left(\frac{1}{H}+R-r\right) - H(R^2-r^2)\int_{R-r}^{R+r}\frac{e^{-Hx}}{x}dx\right]$$

$$= k\pi\delta_v\left[\frac{2R^2}{H} - \frac{2R}{H} + \frac{1}{H^3} - \frac{1}{H^3e^{2HR}} - H\int_0^R r(R^2-r^2)dr\int_{R-r}^{R+r}\frac{e^{-Hx}}{x}dx\right].$$

We can carry out the double integration of the last term, changing the order of integration; but it is necessary to change also suitably the limits of integration. Proceeding in this way, and making $p=RH$, we have finally

$$F = k\pi\delta_v R^3\left[\frac{1}{p} - \frac{1}{2p^3} + e^{-2p}\left(\frac{1}{p^2}+\frac{1}{2p^3}\right)\right]. \quad . \quad (3)$$

The external action of gravitation can therefore be considered as the effect of this flux. Inasmuch as k is the coefficient of proportionality that gives the Newtonian force as a function of the apparent mass, calling this mass M_a, we have

$$F=kM_a\,;\;\; M_a=\pi\delta_v R^3\left[\frac{1}{p}-\frac{1}{2p^3}+e^{-2p}\left(\frac{1}{p^2}+\frac{1}{2p^3}\right)\right]. \quad (4)$$

If we put

$$\psi=\frac{3}{4}\left[\frac{1}{p}-\frac{1}{2p^3}+e^{-2p}\left(\frac{1}{p^2}+\frac{1}{2p^3}\right)\right], \quad . \quad . \quad (5)$$

we have

$$M_a=\frac{4}{3}\pi\delta_v R^3\psi=M_v\psi, \quad . \quad . \quad . \quad . \quad (6)$$

in which M_v represents the true mass of the sphere. Moreover we have

$$M_a=\frac{4}{3}\pi\delta_a R^3\,;\;\; \delta_v=\frac{\delta_a}{\psi}\,;\;\; \delta_a=\delta_v\psi\,;\;\; \psi=\frac{\delta_a}{\delta_v}. \quad . \quad (7)$$

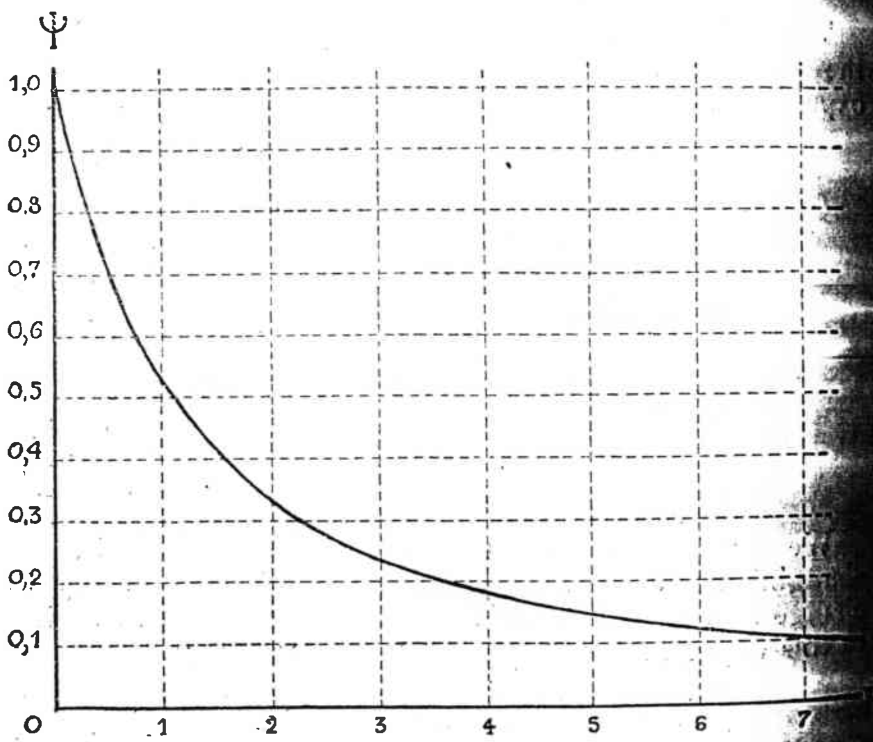
It is easy to see that

$$\lim_{p=0}\psi=1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and hence

$$\lim_{p=0} M_a=M_v.$$

Fig. 2.



That is to say, that the apparent and true mass coincide if $p=0$, that is to say, if it is a question of a sphere with a very small radius, or if $H=0$. In fig. 2 the values of p are down on the abscissa, and the ψ values on the ordinates; in this way the curve corresponding to equation (15) is obtained It touches the axis of the ordinates with a value 1 (see (8)); and it is asymptotic to the axis of the p's.

Application of the ψ function to the Sun.—The sun's density is assuredly not uniform; but for a roughly approximate research I shall suppose this density to be constant, and equal to δ_v. Its apparent density is the astronomical one, and we have $\delta_a=1{\cdot}41$. Several hypotheses can be made as to the real value of δ_v for the sun; for each of them one can calculate the value of ψ by means of (7); then, from the curve of fig. 2, the corresponding value of p can be deduced; and finally, since $p=\mathrm{RH}=\mathrm{R}_s\delta_v h$, we can deduce the value of h, seeing that the sun's radius is $\mathrm{R}_s=6{\cdot}95 \,.\, 10^{10}$ cm. We can construct the following table:—

$\delta_v=1{\cdot}41$	2	5	10	15	20
$\psi=\delta_a/\delta_v=1$	0·705	0·281	0·141	0·094	0·070
$p=0$	0·53	2·46	5·20	7·95	10·40
$p/\mathrm{R}_s\delta_v=0$	$3{\cdot}81.10^{-12}$	$7{\cdot}08.10^{-12}$	$7{\cdot}49.10^{-12}$	$7{\cdot}63.10^{-12}$	$7{\cdot}64.10^{-12}$

Hence we see that if the true density is increasing, the h value increases rapidly, up to a density of about 2; and then more slowly, with a tendency towards a limit-value that we can see remains fixed at $7{\cdot}65 \,.\, 10^{-12}$.

Furthermore, we can see that, even admitting only a true density of the sun slightly greater than the apparent (*i. e.* 2), the order of magnitude for the h *quenching factor* remains fixed between 10^{-12} and 10^{-11}.

The h factor.—According to the already made hypothesis, the h factor represents a *universal constant*, upon which the measure of the gravitational absorption depends; and its probable value would be, as aforesaid, fixed between 10^{-12} and 10^{-11}; but its exact determination cannot be arrived at by considering the solar phenomenon. In fact, we lack the necessary elements to enable us to state the true density of the sun; perhaps we can believe it to be certainly greater than 1·41 (apparent or astronomical density), when we consider the great density of some heavier bodies. The sun's very high temperature, that would have the effect of dilating to an enormous extent such bodies, might be compensated by the enormous pressure in the solar mass interior. Anyway the value of the aforesaid true density of the sun cannot be established *a priori* with sufficient exactitude.

We can then imagine an experimental method for the investigation of the h constant. It would consist in finding the eventual variation of weight for a mass m, relatively small, surrounded by another mass M much bigger. In fact, as it is probable, according to the already made hypothesis, that the flux of the mass m must be partly absorbed by M, so also the gravitational flux that is put forth by the earth must weaken itself, before it reaches m, across M. I suppose this mass M shaped as a sphere, with radius r; and the mass m small, and situated at M's centre. If δ is the density of the substance that forms M, we shall have, according to (1) and (2),

$$f_m = km\,e^{-h\delta r};$$

this represents the flux of the mass m that succeeds in emerging from M. Taking in correspondence m_a and m_v as the true and apparent mass of m, we have

$$\frac{m_a}{m} = e^{-h\delta r}, \text{ or } m_a = m_v e^{-h\delta r};$$

r being very small (at the most a few decimetres), we have

$$m_a = m_v(1 - h\delta r).$$

This means that the mass m would undergo a variation (of diminution) of

$$\epsilon = m_v h\delta r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From this the value of h could be deduced

$$h = \frac{\epsilon}{m_v\delta r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

We can establish what would be the order of magnitude of ϵ, in a possible experiment of this kind. Let us put

$$m = m_v = 1 \text{ kg.}; \ \delta = 13{\cdot}60; \ r = 10 \text{ cm.}$$

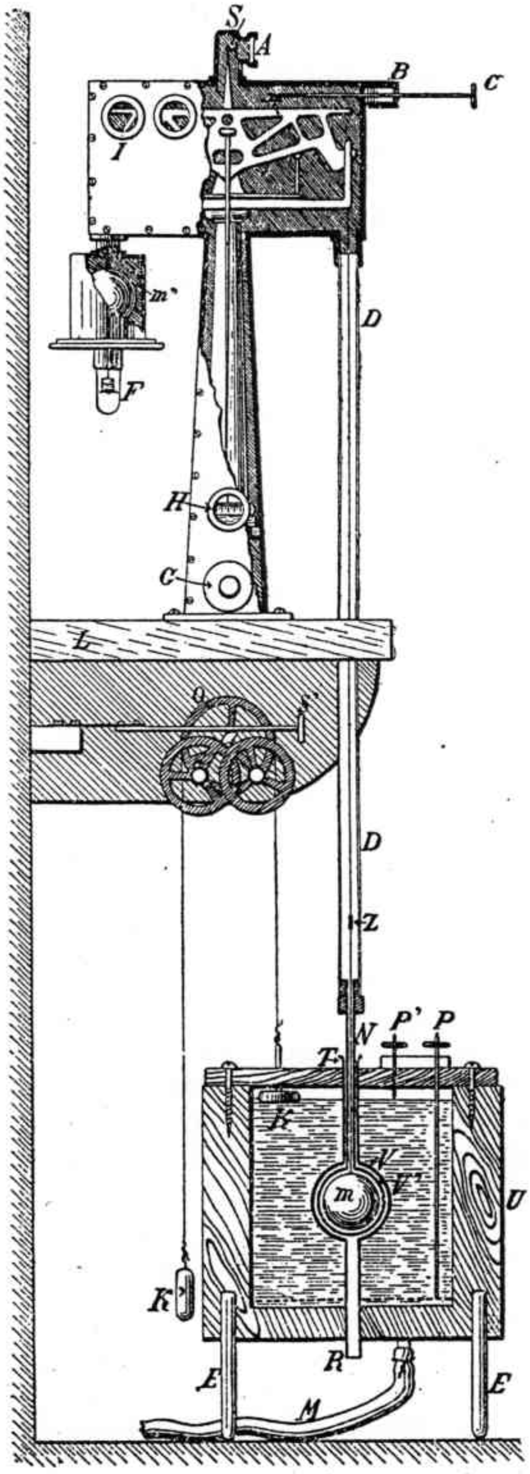
This corresponds to the conditions that I realized in an experiment that I shall describe: I had taken for m a leaden ball; mass M was mercury, symmetrically distributed around m. Since h is probably equal to about 10^{-12} (on the hypothesis that the true density of the sun is a little greater than 2),

$$\epsilon = 1000 \text{ gr.} \,.\, 10^{-12} \,.\, 13{\cdot}60 \,.\, 10 \text{ cm.} = 1{\cdot}4 \,.\, 10^{-7} \text{ gr.}$$

This means that it would be necessary to be able to detect a variation of 1/10,000 of a mg. in 1 kg. The apparatus employed to carry out the planned experiment ought to fulfil such condition.

Description of the experimental disposition.—A Ruprecht balance of the bearing of about 1 kilogram was removed from its original protecting box and enclosed in a metallic box shaped like a T (fig. 3), able to resist the atmospherical

Fig. 3.



pressure when the vacuum was established in it. Special artifices have been devised in order to command from outside the beam and the little rider of a centigram upon it. The scales' original plates also have been taken off. Under the right arm of the balance a glass tube D has been fixed to the protecting box; this tube connects the box itself with the ambient in which is the mass m, as will be described further on. Under the left arm is a metallic protection that contains a leaden ball m', which acts as tare of the mass m: this is also a leaden ball. On the beam at its middle point is a concave mirror S for the observation of the oscillations with a beam of light and vertical scale. The balance and the box are on the bracket table L fixed to the wall.

Under L and on the floor of the room is the recipient U, destined to receive the mercury that will surround the mass m. It is made out of very strong pieces of wood joined together; it is a cylinder whose diameter and inner height are about 22 cm. In the axis of the cylinder U are fixed two brass tubes R, T, of which one is the prolongation of the other, and these are joined by means of a hollow brass sphere V, whose diameter is 79 mm. This sphere can be separated in two pieces by means of a joint with screws in its diametral horizontal plane. In the interior of Y and concentric with it there is a second hollow brass sphere V', whose diameter is 70 mm. It is connected by means of a brass tube N to the glass tube D, that comes down from the balance. The sphere V' and the tube N do not come in contact with any point of the sphere V and tube T. The sphere V' can be separated, like V, into two hemispheres; so that it can contain the leaden ball m. This is suspended, by means of a brass wire, to the right arm of the balance, through the tubes D and N. The enlargement Z of this wire allows us to control with the cathetometer the position of the m ball, relatively to the recipient U. In this the mercury can flow upwards; this liquid can be removed at will by aspiration. The levels that the mercury reaches when U has been filled or emptied nearly completely are controlled rigorously by electric contacts P, P', that can be easily regulated. Moreover, a delicate system consisting of a float K and its counterpoise K' shows, by means of a mirror S', the position that the mercury takes every moment in the recipient U. All the fittings are made with a precision superior to 2/10 of a millimetre; within this limit it can be reckoned that the mercury has its centre of gravity coincident with the centre of gravity of m, the leaden ball. This has a mass of 1274 gr.; the mercury of 104 kg.

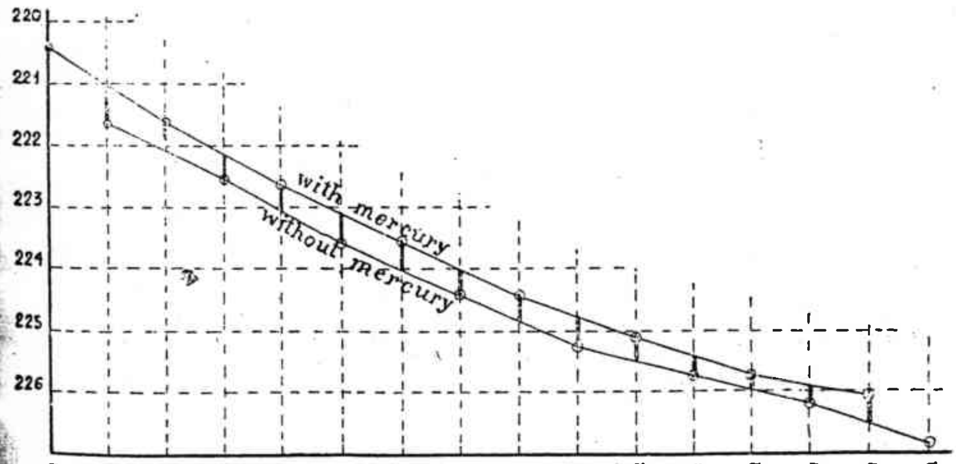
The balance with its accessories maintains the vacuum in a way practically perfect. Even after twenty-four hours the inner pressure does not rise above 7/10 mm. of mercury, which represents the vapour tension of the mastics used to seal the balance. While the tests are going on, the mercury's rotatory pump must be kept going to reduce the pressure to less than 1/10 mm. of mercury. Under such conditions the perturbations of temperature that result from the mercury surrounding the V′ and V capsules are completely avoided.

The observations are made with a beam of light reflected by S on a scale at the distance of 12 m.; it is possible to detect 1/10 mm. on this scale. The sensibility of the balance can be brought in this way to a deviation of 170 mm. for the beam of light *per mg.* Hence it becomes possible to estimate about 1/1700 mg. in a direct reading, and reach a superior preciseness with many observations.

However, a doubt arises, is not such a preciseness illusive, and will not the slightest causes of perturbation hide completely a deviation of a few millimetres? I have nevertheless been able to remove all the more considerable causes of error. The most important of these consisted of the external mechanical shakes produced by the life of the town. I avoided them either by making my observations in the night or by availing myself of the days of the general strike, in this respect very useful.

Observation on the variation of weight.—The effect of the presence of mercury around the ball m was ascertained in the following way. In the recipient U determinations of

Fig. 4.



the balance position of rest were made rapidly and alternately with and without mercury. Fig. 4 shows the diagram of one of this series of observations, carried out on July 20–21, 1919.

As abscissæ I take the successive intervals of time $C_1 S_2$, $S_2 C_3$, $C_3 S_4$, $S_4 C_5$, ..., all equal, elapsing between the single observations *with mercury* and *without mercury* alternately. As ordinates I take the scale's positions of rest, determined each of them with three readings of oscillations. I join the points obtained in this way with two lines. These have a descending direction caused by the gradual displacement of the rest position of the balance, which displacement is a consequence of slight variations in the temperature. But the *with-mercury line* is always, with its points, above the *without-mercury one*. This means that the presence of the mercury *always* makes the leaden ball *m* *seem lighter*. In the same fig. 4 the 14 vertical segments represent the various successive means obtained from the represented series of observations. For briefness sake I shall not report all the other diagrams corresponding to five other series of observations that were made, together with the one described in fig. 4, on July 20 and 21. I shall say only that, taking the general mean of 57 partial means, I find as the value of the displacement of the balance position of rest, due to the presence of the mercury,

$$\text{mm. } 0{\cdot}358 \pm 0{\cdot}012\,;$$

the probable error 0·012 has been calculated by the method of least squares. The direction of the displacement indicates a diminution of weight, that is to say, *absorption of the terrestrial force of gravitation* on the leaden ball through the mercury.

The balance sensibility in the course of the aforesaid experiments was constantly equal to 171 mm. per mg. Therefore that displacement corresponds to a variation of

$$\text{mg. } \frac{0{\cdot}358 \pm 0{\cdot}012}{171} = \text{mg. } 0{\cdot}00209 \pm 0{\cdot}00007.$$

Correction of the observed effect.—However, it must be stated that in the experiment carried out in this way, several causes intervene, and superposing themselves with their own effects upon the phenomenon sought, modify the result notably. I cannot in this brief exposition discuss such causes in detail ; but among them I select those which have sensible effect, and I construct with them the following table (each bears its own sign) :—

Observed effect	+mg. 0·00209±0·00007
Newtonian effect of the Hg on the tare	−mg. 0·00085
„ „ „ receptacle of Hg	+mg. 0·00007
„ „ „ floats K and K′	−mg. 0·00034
„ „ „ Hg on the beam	mg. 0·00000
Correction for the displacement of the zero...	+mg. 0·00001
Greatest error admissible for asymmetry	mg. ±0·00009
Net effect	ϵ=+mg. 0·00098±0·00016

The four corrections given here for the Newtonian effects of the mercury and float K with the K′ counterpoise have been rigorously calculated. Their probable error is very much inferior to the probable error of my observations. The greatest error admissible for asymmetry in the mercury's position relatively to the leaden ball, estimated at ±0·00009 mg., is certainly superior to the true one: it has been my purpose to exaggerate in its admission, to show that it cannot cover the phenomenon that has been discovered.

Hence *we have a net diminution of weight undergone by the leaden ball weighing* 1274 *gr., and this diminution is equal to* 0·00098 *mg.* (*that is to say*, $7{\cdot}7 \cdot 10^{-10}$ *of its value*) *due to the fact of the ball being surrounded by the mercury.*

Possibility of other causes of error.—In the detailed relation of these experiments that will be published by the *Accademia dei Lincei* in Rome, I discuss minutely the possibility of other causes of error. Here I shall simply enumerate them :—

I. Perturbations of a mechanical character, such as the effects of the weight of the mercury on the balance of the projection lamp, the position of the scale, or the deformation of the recipient U, augmentation by compression of the density of the mercury, &c.;
II. Perturbations of a calorific character;
III. Radiometric actions;
IV. Electrostatic actions;
V. Magnetic actions;
VI. Electromagnetic actions.

I shall add only that such causes of error, if intervening, cannot modify sensibly the result that has been obtained.

Determination of the h constant.—The already verified variation of weight allows the valuation of the universal quenching constant h at least between certain limits of approximation. I make use of the relations (9) and (10). I must, however, introduce an hypothetical simplification in the experiment as performed if I want to avoid a very great difficulty of calculation; moreover, for a first research of the kind, it may be permitted. I shall suppose the leaden mass m, weighing 1274 gr. concentrated in one point; I shall suppose, moreover, the mass of mercury, weighing 104 kg. transformed from cylindrical to spherical shape, though containing still concentrically the V sphere (fig. 3). The radius of the mercury sphere so resulting will be equal to 12·35 cm. Finally, the thickness of mercury crossed by the single gravitational actions that the lead puts forth (or receives) can be rated, always in rough approximation, as equal to the difference between the radius of the mercury sphere and the sphere V. This corresponds to $12{\cdot}35-3{\cdot}95=8{\cdot}40$ cm. Consequently we have in the formula (10):

$$\epsilon=9{\cdot}8.10^{-7}\text{ gr.};\ m_v=1274\text{ gr.};\ \delta=13{\cdot}60\ ;\ r=8{\cdot}40;$$

and hence

$$h=\frac{9\,.\,10^{-7}}{1274\,.\,13{\cdot}60\,.\,8{\cdot}4}=6{\cdot}73\,.\,10^{-12}.$$

The order of magnitude in this determination coincides with the one predicted.

Application of the experimental result to the Sun's case.—The results already obtained are based chiefly upon the hypothesis that the sun's astronomical density, called here *apparent*, may be inferior to another density: the true density. Making always the simplification, deriving from the hypothesis of the constancy of the sun's true density, it can be considered as determined by the experiment described above. Let us call R_s the sun's radius, δ_{rs}, δ_{as}, its densities (apparent and true). Putting $p=RH=Rh\delta$, we have for the sun

$$p_s=h\delta_{vs}R_s.$$

To p_s's value corresponds a determinate value ψ_s of the ψ function, which might be deduced from fig. 2, if δ_{vs} were known to us. Now, from (7), we have

$$\delta_{vs}=\frac{\delta_{as}}{\psi_s};$$

therefore

$$p_s\psi_s=hR_s\delta_{as}.$$

Since $R_s=6{\cdot}95\,.\,10^{10}$ cm. $\delta_{as}=1{\cdot}41$; $h=6{\cdot}73\,.\,10^{-12}$; we have further

$$p_s\psi_s=6{\cdot}18\,.\,10^{-12}\,.\,6{\cdot}95\,.\,10^{10}\,.\,1{\cdot}41=0{\cdot}660.$$

This condition must be satisfied. Considering the curve in fig. 2 we note that for the point $p=2{\cdot}0$, $\psi=0{\cdot}433$, and this really does happen. Hence I will put $p_s = 2\cdot0$, $\psi_s = 0\cdot433$, and from this we can deduce

$$\delta_{vs} = \frac{\delta_{as}}{\psi_s} = \frac{1\cdot41}{0\cdot433} = 4\cdot27.$$

That is to say, *it ensues that the sun's true density is the triple of what it is believed to be by the astronomers* (1·41).

But, although considering it possible to take as true the general result about a true density greater than the apparent, I do not attach exceeding importance to the determination I have been working out. The problem on the research of the true density so laid down, shows itself to be rather uncertain. In fact, it is sufficient to admit even a relatively slight error in the determination of ϵ, to have the value of δ_{vs} notably altered. That can be deduced from the following table:—

ϵ.	h.	δ_v.
0·0007	$4\cdot80 \,.\, 10^{-12}$	2·42
0·0009	$6\cdot18 \,.\, 10^{-12}$	3·27
0·00098	$6\cdot73 \,.\, 10^{-12}$	6·73 (experimental determination).
0·0011	$7\cdot55 \,.\, 10^{-12}$	10·04
0·0012	$8\cdot23 \,.\, 10^{-12}$	——

i. e., it is enough to admit $\epsilon = 0\cdot0011$, to have the true density rise to 10·04.

But, considering the ψ function, we come to note an interesting consequence: the h constant cannot be greater than $7\cdot55 \,.\, 10^{-12}$; because if it were, in the sun's case the following expression ought to be exact:

$$\frac{p_s\psi_s}{R_s\delta_{as}} > 7\cdot65 \,.\, 10^{-12}, \text{ or } p_s\psi_s > 0\cdot75,$$

and such a condition never can be verified from (5), which for great values of p, gives at most $p\psi = \frac{3}{4}$.

In other words we can also say *inasmuch as the sun has an apparent density equal to* 1·41, *the coefficient of absorption h cannot be greater than* $7\cdot65 \,.\, 10^{-12}$. The experiment gives $6\cdot73 \,.\, 10^{-12}$; therefore the facts agree up to now with the proposed theory.

I bring these considerations to a close, calling attention to the fact that if we admit the hypothesis of gravitational absorption, the calculation worked out for the sun with the simplification of constant density cannot lead us to very erroneous results. In fact, if we substitute for this hypothesis of constant density, another law of variable density, this will be, as a matter of course, greater at the centre than at the surface. Therefore, on the one hand, the fact that the matter would accumulate itself towards the centre would have for consequence that the absorption of the gravitational force of the greatest part of the matter would be accomplished across greater thickness, as the gravitational action would have to pass first chiefly from the deeper layers to the surface, then to the exterior ; but, on the other hand, the exterior mass has a reduced density, hence the absorption itself is diminishing. Therefore these are two contrary causes that in general will not balance each other, but the effect of one is subtracted from the effect of the other, leaving the mean true density not very different from the one established by my experiment and calculation.

Summary and Conclusion.—Examining Newton's Law, I have come to think that the force of gravitation can weaken itself by absorption due to ponderable matter. Following other arguments, I have come to suspect that the matter which shows the force of gravitation might heat itself. Although such conception might resolve in a new way the old controversy on the origin of the sun's heat, I state it with all reserve.

I have undertaken afterwards to treat theoretically the case of a spherical mass with constant density, subject to the absorption of its own force of gravitation, and from this work I have deduced the elements necessary to carry out an experimental control of my hypothesis. I have carried out this experiment by weighing *in vacuo* a leaden ball whose weight was 1274 gr. symmetrically surrounded by 104 kg. of mercury. Having previously avoided all the possible causes of error, I have been able to conclude that *the leaden ball loses* $7{\cdot}7 \,.\, 10^{-10}$ *of its weight by the presence of the mercury.* Such result causes the determination of the *quenching constant* (factor of absorption) per unit of density and length, as $6{\cdot}73 \,.\, 10^{-12}$.

Applying finally this result to the sun's case, I calculate its true density as 4·27.

The importance of this research is obvious, and I do not think that reasons for criticism can easily be found. Anyway, as I am the first to wish to test in all possible ways the results I am publishing, I may mention that it is my intention to repeat my experiments with far bigger apparatus. For the purpose, in the Laboratorio di Fisica del Politecnico di Torino (Italy), an apparatus is being built in such proportions as will render possible an experiment with 10,000 kg. of lead. On the results that I shall obtain with it I shall report in due time.

XLIX. *The Hot-wire Anemometer: its Application to the Investigation of the Velocity of Gases in Pipes.*

By J. S. G. THOMAS, *M.Sc.(Lond.), B.Sc.(Wales), A.R.C.S., A.I.C.**

[Plates X.–XIII.]

THE possibility of utilizing the cooling effect experienced by a fine heated platinum wire, when immersed in a stream of fluid, as a method of practical anemometry has been placed on a sound theoretical basis by King †. Morris ‡ has examined the characteristics of wires of various kinds for use in this connexion, and has described a number of methods of employing the hot wire for the same purpose. Both these investigators § and others ‖ are responsible for types of so-called hot-wire anemometers to be employed for the measurement of the velocity of air-currents. The author has recently had occasion to examine the possibility of the use of such instruments in an investigation connected with the flow of gases through pipes and orifices, upon which he is at present engaged, and the present paper contains an account of certain interesting results obtained as the result of such examination. Of the various types of hot-wire anemometers available, the type due to Morris and described by him in Eng. Pat. 25,923/1913, was found on examination to be the most suitable for the purpose of the investigation. This type of anemometer is constituted of four equal wires of the same material—platinum by choice—composing the four separate arms of a Wheatstone bridge. One pair of alternate arms of the bridge is shielded by means of surrounding tubes; the resistances being all adjusted to equality initially at any temperature, the bridge remains balanced at any other temperature. The bridge-wires being inserted into a stream of fluid, the balance of the bridge is upset, the unshielded arms alone being subjected to the cooling effect of the fluid current, and the galvanometer deflexion serves, after calibration of the instrument, to measure the velocity of the stream. Morris, in the calibration of his instruments, employed air-currents produced in a vertical wind channel, the stream of air passing vertically downwards therein, at

* Communicated by the Author.

† Phil. Trans. Roy. Soc., A. 520, 214, pp. 373-432 (1914). Phil. Mag., 1915, p. 570.

‡ British Association, Dundee, 1912; Electrician, Oct. 4, 1912, p. 1056; Engineering, Dec. 27, 1912.

§ King, Eng. Pat. 18,563/1914; Morris, Eng. Pat. 25,923/1913.

‖ See *e. g.* King, Phil. Trans., *loc. cit.* p. 404.